interior Green's function approach of [6] must be used. In this case, however, there is no exterior S matrix contribution, only an additional set of conditions on the picture-frame nodes.

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Hall Field and Magnetoresistance Effects in Rectangular Waveguide Completely Filled with Semiconductor

J. B. NESS AND M. W. GUNN

***Abstract*—Microwave propagation through a rectangular waveguide completely filled with semiconductor and subject to a transverse magnetic field is analyzed. When the magnetic field is parallel to the broad wall of the waveguide (the x axis), propagation is analyzed in terms of the Hall effect. For the magnetic field parallel to the y axis, the effect of the field on the propagation is shown to be due to longitudinal magnetoresistance effects. Good agreement is obtained between theory and experiment in both cases. The experiments were performed at 30 GHz using n-type germanium.**

Introduction

In this short paper the effects of the application of transverse magnetic fields on the propagation coefficient of a rectangular waveguide completely filled with semiconductor (Fig. 1) are considered theoretically and experimentally. In the absence of the magnetic field the dominant propagation mode in the semiconductor-filled waveguide will be the TE_{10} mode. However, the semiconductor permittivity becomes a tensor in the presence of a magnetic field and the tensor permittivity causes coupling of higher order modes to the TE_{10} mode. It has been shown previously [1] that propagation in the presence of a magnetic field in the x direction is by TE_{0n} modes or by anomalous modes having all six field components.

The method of analysis used here is an approximation technique based on Schelkunoff's "generalized Telegraphists Equation" and adopted previously to analyze the partially filled guide [2]. The propagation characteristics are analyzed in terms of coupling between modes which implies that the propagation mode in the presence of a magnetic field along the x axis is by an anomalous mode rather than TE_{0n} modes.

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The authors are with the Department of Electrical Engineering, University of Queensland, Brisbane 4067, Qld., Australia.

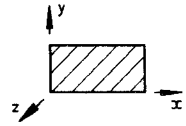


FIG I FULLY FILLED GUIDE

Fig. 1. Fully filled guide.

Since the tensor permittivity is derived assuming that the semiconductor has spherical constant energy surfaces, this method predicts that propagation will be unaffected by a magnetic field directed along the y axis, as the magnetic field is now parallel to the electric field of the TE_{10} mode. However, experimentally the germanium samples do show a magnetic-field dependence for the field along the y axis and the affect is explained qualitatively and quantitatively by the longitudinal magnetoresistance effect.

Experiments performed at 30 GHz using a microwave transmission bridge are used to verify the theoretical analyses for both directions of magnetic field. At this frequency the results indicate that the effects of carrier inertia are measurable, and by taking the relaxation time into consideration better agreement between theory and experiment is obtained.

Theory

Consider an electromagnetic wave propagating through a semiconductor. The total current density can be written as

$$\boldsymbol{J} = \boldsymbol{J}_d + \boldsymbol{J}_c \tag{1}$$

where

$\boldsymbol{J}_d$ the displacement current density;
$\boldsymbol{J}_c$ the conduction current density.

In the presence of a magnetic field the semiconductor permittivity becomes a tensor so that the current density can now be written as

$$\boldsymbol{J} = [\varepsilon] \frac{\partial \boldsymbol{E}}{\partial t}. \tag{2}$$

The particular form of the permittivity tensor will depend on the assumptions used in the derivation. Engineer and Nag [1] have developed a form for this tensor by including the Hall field in Maxwell's equations, although all terms were assumed frequency independent. Kataoka and Fujisada [3] have obtained expressions for the tensor permittivity terms using the basic equations of motion although the lattice permittivity term was neglected in this derivation. The following derivation based on the equation of motion for electrons under the influence of an applied alternating electric field and a steady magnetic field [4], includes both of these factors. Thus

$$m_e{}^* \frac{d\bar{v}_e}{dt} + m_e{}^* \frac{\bar{v}_e}{\tau_e} = -q(\boldsymbol{E}e^{j\omega t} + \bar{v}_e \times \boldsymbol{B}) \tag{3}$$

where

$\boldsymbol{B}$ the magnetic-flux density;
τ_e the relaxation time for electrons assumed isotropic and constant;
$\bar{v}_e$ the average induced velocity of the electrons;
$m_e{}^*$ the effective mass of the electrons.

For electromagnetic propagation along the z direction and with the magnetic field in the x direction, (3) can be written in component form as

$$\frac{d\bar{v}_{ex}}{dt} + \frac{\bar{v}_{ex}}{\tau_e} = \frac{-q}{m_e^*} E_x e^{j\omega t}$$

$$\frac{d\bar{v}_{ey}}{dt} + \frac{\bar{v}_{ey}}{\tau_e} = \frac{-q}{m_e^*} E_y e^{j\omega t} - \frac{q}{m_e^*} B_x \bar{v}_{ez}$$

$$\frac{d\bar{v}_{ez}}{dt} + \frac{\bar{v}_{ez}}{\tau_e} = \frac{-q}{m_e^*} E_z e^{j\omega t} + \frac{q}{m_e^*} B_x \bar{v}_{ey}. \quad (4)$$

The solution of these equations together with (1) and (2) gives the following expressions for the tensor terms

$$\varepsilon_{11} = \varepsilon\left[1 - j\frac{\sigma}{\omega\varepsilon}\right]$$

$$\varepsilon_{12} = \varepsilon_{13} = \varepsilon_{21} = \varepsilon_{31} = 0$$

$$\varepsilon_{22} = \varepsilon_{33} = \varepsilon\left[1 - \frac{j\sigma}{\omega\varepsilon(1 + \mu^2 B_x^2)}\right]$$

$$\varepsilon_{23} = -\varepsilon_{32} = \frac{j\sigma\mu B_x}{\omega(1 + \mu^2 B_x^2)} \quad (5)$$

where

$$\sigma = \frac{\sigma_0}{1 + j\omega\tau_e}$$

$$\mu = \frac{\mu_0}{1 + j\omega\tau_e}$$

and

σ_0 the dc conductivity;
μ_0 the dc mobility for electrons;
ε the lattice permittivity.

The relaxation time τ_e, assumed isotropic and independent of energy, was calculated as 7×10^{-13} s from the doping density and effective mass of the carriers [4]. The lattice permittivity, like the conductivity, is frequency and temperature dependent, but the correction resulting from the relaxation time is a second-order term for the lattice permittivity. It is negligible except at low temperatures where the relaxation time is increased [5] and has been neglected here. The further assumption is made that the mobility is independent of magnetic field. This is a reasonable assumption for n-type germanium subject to a maximum magnetic-flux density of 2.0 T [7].

When the magnetic field is applied in the y direction the analysis is considerably simplified. In this case, the magnetic field is parallel to the electric field of the incident TE_{10} mode. The Hall field is not present and the influence of the magnetic field on propagation is due to longitudinal magnetoresistance effects. The TE_{10} mode will be undistorted by the magnetic field, but the propagation coefficient will be altered by the magnetic-field dependency of the conductivity. The conductivity can be measured as a function of magnetic field using dc techniques. The propagation coefficient is then simply that for a fully filled guide and is readily calculated for each value of magnetic field.

The propagation coefficient for the loaded waveguide is found using the approximation technique previously described [2] with the fields in the semiconductor-filled waveguide approximated by the coupling between the TE_{10}, TE_{11}, and TM_{11} modes. When three modes are selected the equation for the propagation coefficient γ is a sixth-order polynomial. That is

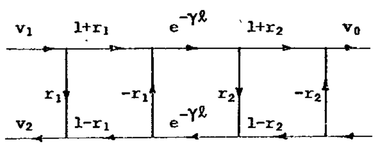
$$\gamma^6 + A_5\gamma^5 + A_4\gamma^4 + A_3\gamma^3 + A_2\gamma^2 + A_1\gamma + A_0 = 0. \quad (6)$$

For this fully filled waveguide, propagation is reciprocal and the coefficients of the odd powers of γ are zero. Equation (6) reduces to

$$\gamma^6 + A_4\gamma^4 + A_2\gamma^2 + A_0 = 0. \quad (7)$$

The numerical solution of (2) gives three propagation coefficients corresponding to the distorted TE_{10}, TM_{11}, and TE_{11} modes. When the field is zero, the TE and TM modes can propagate as undistorted modes in the fully filled section and the TE_{10} mode will have the lowest attenuation coefficient. Since the TE_{10} mode is the incident mode, higher order TE and TM modes will not be readily launched in the fully filled guide even when a magnetic field is applied. Thus the root with the lowest attenuation coefficient corresponds to the TE_{10} mode and is the propagating mode.

For single-mode reciprocal propagation the loaded section of waveguide can be represented by the signal flow graph shown [6].



l the length of loaded section;
r_1 the reflection coefficient of the front interface;
r_2 the reflection coefficient of the back interface;
γ the propagation coefficient of the loaded section.

The transmission coefficient is then given by

$$T = \frac{v_0}{v_1} = \frac{(1 + r_1)(1 + r_2)e^{-\gamma l}}{1 + r_1 r_2 e^{-2\gamma l}} \quad (8)$$

reflection symmetry gives $r_1 = -r_2$,

$$\therefore T = \frac{(1 - r_1^2)e^{-\gamma l}}{1 - r_1^2 e^{-2\gamma l}}. \quad (9)$$

Since the expression for T contains two unknowns γ and r_1 a single measurement of T does not permit the propagation coefficient γ to be determined. However, if measurements are made on two samples, identical except for length, the value r_1 will be the same and the transmission coefficients can be written

$$T_1 = \frac{(1 - r_1^2)e^{-\gamma l_1}}{1 - r_1^2 e^{-2\gamma l_1}}$$

$$T_2 = \frac{(1 - r_1^2)e^{-\gamma l_2}}{1 - r_1^2 e^{-2\gamma l_2}}. \quad (10)$$

Writing r_1 in terms of T_1 and T_2 in the previous equations and equating the results, it may be shown that

$$\frac{T_1 - e^{-\gamma l_1}}{(e^{-\gamma l_1} - T_1 e^{-2\gamma l_1})} = \frac{T_2 - e^{-\gamma l_2}}{(e^{-\gamma l_2} - T_2 e^{-2\gamma l_2})} \quad (11)$$

which can be written

$$T_1(e^{\gamma l_1} - e^{-\gamma l_1}) + T_2(e^{-\gamma l_2} - e^{\gamma l_2}) + T_1 T_2(e^{-\gamma(l_1 - l_2)} - e^{-\gamma(l_2 - l_1)}) = 0. \quad (12)$$

A numerical solution of this equation together with the measured

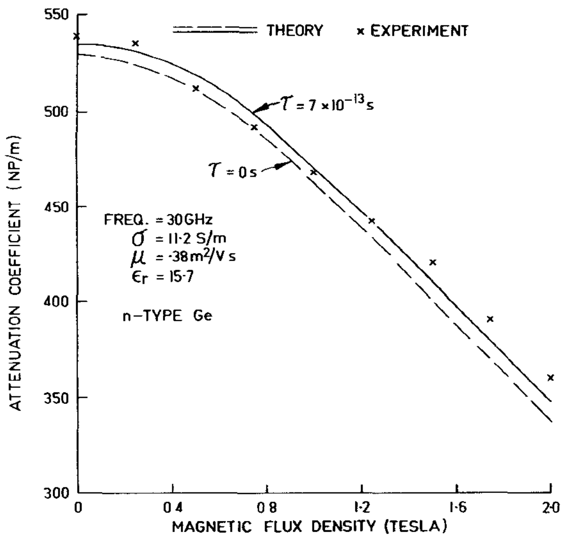


Fig. 2. Variation of attenuation coefficient with magnetic-flux density. (Magnetic field parallel to x axis.)

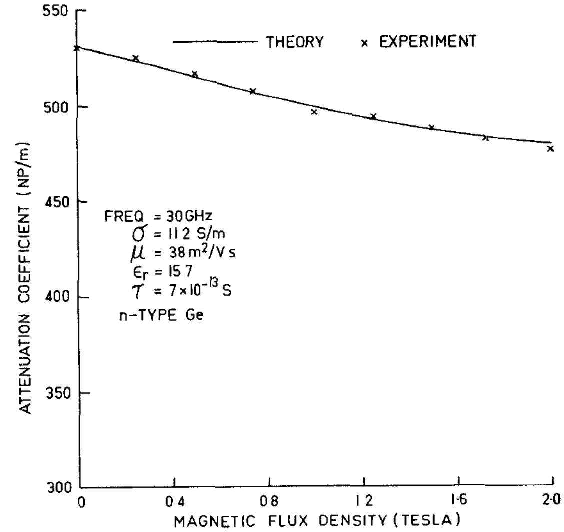


Fig. 4. Variation of attenuation coefficient with magnetic-flux density. (Magnetic field parallel to y axis.)

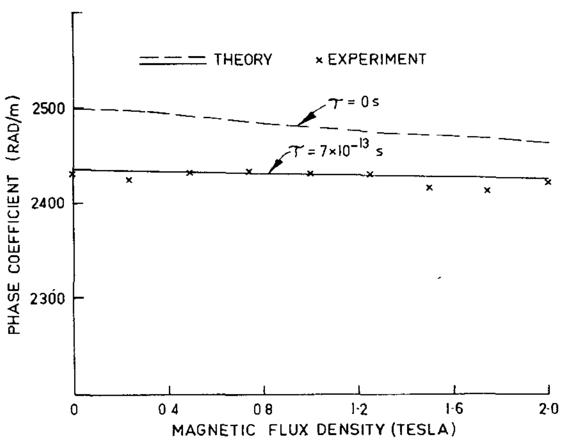


Fig. 3. Variation of phase coefficient with magnetic-flux density. (Magnetic field parallel to x axis.)

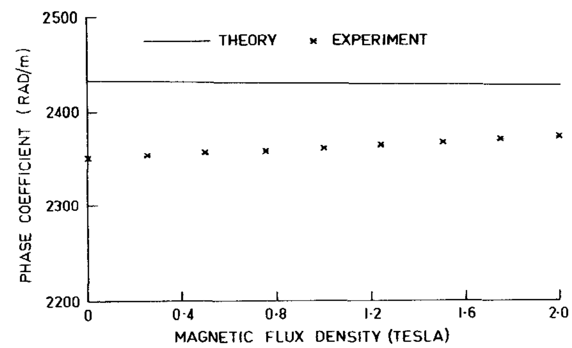


Fig. 5. Variation of phase coefficient with magnetic-flux density. (Magnetic field parallel to y axis.)

values T_1 and T_2 enables the propagation coefficient γ to be determined.

For the simple case of a fully filled guide in the absence of any applied magnetic fields, (9) reduces to the well-known form

$$T = \frac{4\gamma\gamma_0}{(\gamma + \gamma_0)^2 e^{\gamma l} - (\gamma - \gamma_0)^2 e^{-\gamma l}} \tag{13}$$

where γ_0 is the propagation coefficient for the TE_{10} mode in the empty guide. In this case, the propagation coefficient can be evaluated from a single measurement of the transmission coefficient.

Results

Fig. 2 shows the theoretical and measured attenuation coefficient as a function of magnetic field for the fully filled guide with the magnetic field applied along the x axis. The value of conductivity was measured as 11.2 S/m at dc and the relative permittivity was taken as 15.7 [5]. The theoretical and experimental values of the propagation coefficient as a function of magnetic field are shown in Figs. 2 and 3. The agreement between theory and experiment is good. The effect of frequency on the conductivity is indicated by the inclusion of curves for $\tau = 0$. For large values of magnetic field the experimental values of the attenuation coefficient tend to diverge somewhat from the theoretical values. This could be due to the slight dependency of the mobility on the magnetic field as well as the increasing inaccuracy of the approximation analysis, as the TE_{10} mode becomes more distorted with increasing magnetic field.

A steady magnetic field applied in the y direction should have no effect on the propagation coefficient of the TE_{10} mode if the semiconductor has spherical constant-energy surfaces. However, germanium has ellipsoidal energy surfaces and exhibits longitudinal magnetoresistance, which is the change in resistance with magnetic field when this field is parallel to the electric field. The dc conductivity was measured as a function of magnetic field and these values were used to compute the theoretical curves shown in Figs. 4 and 5. The experimental results obtained when the magnetic field was applied along the y axis are also shown on the same figures. The agreement between theory and experiment is again quite good, the maximum error being about 3 percent. When the magnetic field is zero the experimental phase coefficient is found to be somewhat smaller than in the previous case. This is caused by the slight alteration in the air-gap distribution when the waveguide is rotated through 90°. The effect of the air gap on the attenuation coefficient is not as marked. The results confirm that the change in the propagation coefficient is due to the longitudinal magnetoresistance effect. The results also show that the longitudinal magnetoresistance effect at 30 GHz is of the same order as the dc value. This was expected from previous investigations [8], [9] which showed the frequency independence of transverse magnetoresistance effects.

Experimental Details

The experimental results were obtained using a microwave transmission bridge operating at 30 GHz and carefully adjusted to avoid internal reflections within the bridge. The semiconductor samples were cut from a single crystal block of germanium using a diamond saw and then lapped to the required dimensions. The internal dimensions of the waveguide are 7.112×3.556 mm $\pm$ 0.02 mm and the samples were lapped to 7.05×3.53 mm to enable them to be inserted and removed from the waveguide without chipping or cracking. The samples were lapped to size on fine (1000) carborundum paper and the front and back faces were then polished with a fine aluminum oxide powder. The samples were then quickly etched in an acid solution and washed in acetone and distilled water.

The effect of the air gap between the sample and the waveguide walls has been discussed in [5] and correction terms are given. For this case the correction terms are small and were reduced further by coating the sides of the sample in contact with the waveguide walls with a highly conducting silver epoxy. However, the air gap still had a slight but noticeable effect on the phase coefficient. This effect is larger when the air gap is distributed equally at both the broad walls of the guide rather than when one side of the sample is in complete contact with the broad wall.

DC measurements of the conductivity and magnetoresistivity were made with rectangular specimens suitably etched to remove any higher conductivity surface layers and leads were soldered to the samples using Sn/Sb solder.

Since single crystal samples were used, it was necessary to ensure that the current and magnetic-field directions retained the same orientation with respect to the crystal axes for both the dc and microwave measurements. The direction of microwave propagation was always taken along the $\langle 111 \rangle$ crystal axis.

Conclusion

The effect of a steady magnetic field on microwave propagation through a fully filled semiconductor-loaded waveguide has been analyzed. When the magnetic field is perpendicular to the electric field of the incident TE_{10} mode, the propagation can be explained in terms of the distortion of the TE_{10} mode by the Hall effect. The simple theory of spherical constant-energy surfaces is shown to give good results in this case. Although the transverse magnetoresistance effect is implicit in this method, this effect itself does not adequately account for the magnetic-field dependency of the propagation coefficient. When the magnetic field is parallel to the incident electric field, the TE_{10} mode is the propagating mode and the effect of the magnetic field on the propagation coefficient is explained by longitudinal magnetoresistance effects. The results indicate that magnetoresistance is frequency independent for frequencies at least as high as 30 GHz.

The relaxation time τ has also been shown to affect the propagation coefficient at this frequency, although it would be difficult to estimate the relaxation time from these measurements. To enable τ to be measured in this way the conductivity and dielectric constant would need to be known very accurately. Experimental problems, such as sample inhomogeneity and the limited accuracy of commercially available microwave components, would also need to be overcome. However, this method of measurement provides a simple means of measuring the Hall mobility of semiconductors at microwave frequencies.

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Propagation Along a Braided Coaxial Cable Located Close to a Tunnel Wall

DAVID A. HILL, SENIOR MEMBER, IEEE, AND
JAMES R. WAIT, FELLOW, IEEE

***Abstract*—A previous development is extended to permit attenuation calculations when a braided cable is located close to a tunnel wall. This is an important case in mine communications utilizing leaky feeders. Numerical results are presented to illustrate the effects of numerous parameters on mode attenuation. A principal finding is that the attenuation rate for the bifilar mode is hardly affected at all by the finite conductivity of the wall. On the other hand, the monofilar mode suffers a very high attenuation when the cable approaches the wall.**

Introduction

The leaky-feeder technique is now being developed for communication in mines [1]. In this method, referred to as continuous-access guided communications (CAGC), the signals are guided by some type of transmission line. The energy is coupled into or out of the channel by antennas in the vicinity of the transmission line which may be a coaxial cable [2] or a twin-wire line [3], [4].

We have previously derived a mode equation for a braided coaxial cable within a circular tunnel and we presented some numerical results [5]. However, that mode equation is very poorly convergent when the cable is located close to the tunnel wall. Unfortunately, it is precisely this case which is of most practical interest for communication in coal mines where it is generally necessary to lay the cable close to the wall [6]. In this

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D. A. Hill is with the Institute for Telecommunication Sciences, U.S. Department of Commerce, Boulder, CO 80302.
J. R. Wait is with the Environmental Research Laboratories, U.S. Department of Commerce, Boulder, CO 80302, and is a Consultant to the Institute of Telecommunication Sciences.